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On the Phase and Envelope Distributions of Amplitude-Modulated Signals

Stephen H. Sung and Yifeng Zhou

Defence R&D Canada – Ottawa
TECHNICAL MEMORANDUM
DRDC Ottawa TM 2004-037
March 2004

Canada

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Stephen H. Sung
Calian Ltd.

Yifeng Zhou
Defence R&D Canada - Ottawa

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Abstract

In this memo, the probability density functions (pdfs) for the phase and the envelope of an amplitude-modulated (AM) signal, where the modulation is modeled as a Gaussian random process, in the presence of narrow-band Gaussian noise are calculated. An exact, closed-form expression is obtained for the phase pdf, while approximate results, accurate for small and large carrier-to-noise ratios (CNRs), are derived for the envelope pdf.

Résumé

Dans le présent document, on établit les fonctions de densité de probabilité (fdp) de la phase et de l'enveloppe d'un signal modulé en amplitude (AM), dont la modulation est modélisée sous forme d'une fonction aléatoire gaussienne, en présence de bruit gaussien à bande étroite. Une expression analytique exacte est obtenue pour la fdp de la phase, alors que des résultats approximatifs, précis pour des rapports porteuse/bruit faibles et élevés, sont obtenus pour la fdp de l'enveloppe.

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Executive summary

Amplitude modulation (AM) is one of the most common modulation types in civilian and military communications applications. It plays an important role in electronic support measures (ESM), electronic intelligence (ELINT) and communication intelligence (COMINT). In this memo, the probability density functions (pdfs) of the envelope and the phase of AM signals, where the modulation is assumed to be a Gaussian process, are investigated. The derived results can be used in the following applications. The first is the identification of intercepted AM communication signals based on the statistical properties of their envelope and phase. The second is the modeling of intercepted radar or sonar signals as having random amplitudes even though the transmitted signals are of constant amplitude. This random amplitude accounts for "multiplicative noise," caused by, for example, dispersion in the propagation medium. In these two applications, the statistical properties of the envelope and the phase of the signals have not been well treated due to a lack of the appropriate pdfs. In this study, a closed-form pdf for the phase is derived as a function of the carrier-to-noise ratio (CNR) and the signal parameters. For the envelope pdf, approximate expressions are obtained for small and large CNRs. These approximate pdfs are compared with the numerically-integrated results.

Stephen H. Sung, Yifeng Zhou. 2004. On the Phase and Envelope Distributions of Amplitude-Modulated Signals. DRDC Ottawa TM 2004-037. Defence R&D Canada – Ottawa.

Sommaire

La modulation d'amplitude (AM) est un des types de modulation les plus couramment utilisés dans les applications des communications civiles et militaires. Elle joue un rle important dans les mesures de soutien électronique (MSE), le renseignement électronique (ELINT) et le renseignement sur les communications (COMINT). Dans le présent document, on étudie les fonctions de densité de probabilité (fdp) de l'enveloppe et de la phase de signaux AM pour lesquels on suppose que la modulation est un processus gaussien. Les résultats obtenus peuvent être utilisés dans les applications mentionnées ci-après. La première consiste à identifier, en se basant sur les propriétés statistiques de leur enveloppe et de leur phase, des signaux de communication AM interceptés. La deuxième consiste à modéliser, sous forme de signaux d'amplitude aléatoire, des signaux radar ou sonar interceptés qui ont néanmoins été émis avec une amplitude constante. Cette amplitude aléatoire tient compte du "bruit multiplicatif" causé, par exemple, par la dispersion dans le milieu de propagation. Dans ces deux applications, les propriétés statistiques de l'enveloppe et de la phase des signaux n'ont pas été traitées correctement, en raison de l'absence de fdp appropriées. Dans la présente étude, on établit une fdp analytique pour la phase en fonction du rapport porteuse/bruit et des paramètres des signaux. Dans le cas de la fdp de l'enveloppe, des expressions approximatives sont obtenues pour des rapports porteuse/bruit faibles et élevés. Ces fdp approximatives sont comparées aux résultats obtenus par intégration numérique.

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1. INTRODUCTION

Amplitude modulation is one of the most common modulation types in civilian and military communications applications. It also plays an important role in electronic support measures (ESM), electronic intelligence (ELINT) and communication intelligence (COMINT) [2]. In this memo, the pdfs of the envelope and phase of AM signals are investigated, where the modulation is assumed to be Gaussian. The results can be useful in the following two applications. The first application is the exploration of statistical properties of envelope and phase of AM signals for signal classification and identification [3]. The second application is for modeling radar and sonar signals as having random amplitudes even though the original signal is constant amplitude [4][5]. The random amplitude is caused by dispersion in the medium and is often referred to as "multiplcative noise". The random amplitude model is considered to be an appropriate model in radar and sonar applications because it represents the real world [6]. However, because of the lack of appropriate representation for the pdfs of the envelope and phase of an AM in the literature, statistical properties of the envelope and phase of an AM have not been well treated or just ignored. An exact solution appears to be difficult. However, a good approximation would be useful for many applications. In this memo, a closed-form solution for the phase is derived as a function of the carrier-to-noise ratio (CNR). For the envelope pdf, closed-form solutions are obtained for different ranges of CNR values, which correspond to the cases of low and high CNR, respectively. Computer simulations are carried out to verify the results.

2. SIGNAL AND NOISE MODELS

The signal plus noise, $e(t)$, is

$$e(t) = s(t) + n(t). \tag{1}$$

The signal $s(t)$ is modeled as

$$s(t) = A[1 + mg(t)]\cos 2\pi f_c t, \tag{2}$$

where A is the carrier amplitude, m, $0 \leq m < 1$, denotes the modulation index, $g(t)$ the modulating function, f_c the carrier frequency, and t the time variable. The modulating function is assumed to be zero-mean Gaussian, with a variance of σ_m^2. The samples of speech signals can be modeled by non-stationary Gaussian processes [7] while the random amplitude in radar and sonar applications is usually described by stationary or nonstationary zero-mean Gaussian processes [6]. The narrow-band noise, $n(t)$, is modeled as [1]

$$n(t) = x(t)\cos 2\pi f_c t - y(t)\sin 2\pi f_c t, \tag{3}$$

where the quadrature components $x(t)$ and $y(t)$ are independent, identically distributed zero-mean Gaussian processes of variance σ^2. They are statistically independent of $g(t)$.

3. PROBABILITY DENSITY DERIVATIONS

The signal plus noise can be written as

$$e(t) = r(t)\cos[2\pi f_c t + \phi(t)], \tag{4}$$

where the envelope, $r(t)$, and the phase, $\phi(t)$, are given by

$$r(t) = \sqrt{\xi^2(t) + y^2(t)} \quad \text{and} \quad \phi(t) = \arctan\frac{y(t)}{\xi(t)}. \tag{5}$$

In (5), $\xi(t) = A + Amg(t) + x(t) \equiv A + z(t)$. From the assumptions regarding the Gaussian nature and the mutual independence of $g(t)$, $x(t)$ and $y(t)$, $z(t)$ is zero-mean Gaussian with variance

$$\sigma_z^2 = A^2 m^2 \sigma_m^2 + \sigma^2 = (2\rho m^2 \sigma_m^2 + 1)\sigma^2 \tag{6}$$

and is independent of $y(t)$. In (6), $\rho \equiv A^2/(2\sigma^2)$ is the CNR. For ease of exposition, c and a are introduced to be

$$c = 2\rho m^2 \sigma_m^2 \quad \text{and} \quad a = c + 1, \tag{7}$$

and the time variable is henceforth omitted. Since z and y are independent, the joint pdf of ξ and y, $p(\xi, y)$, can be written as

$$p(\xi, y) = \frac{1}{2\pi\sqrt{a}\sigma^2}\exp\left[-\frac{(\xi - A)^2}{2a\sigma^2} - \frac{y^2}{2\sigma^2}\right]. \tag{8}$$

The joint pdf of the envelope and the phase, $p(r, \phi)$, is obtained from the relation $p(r, \phi)drd\phi = p(\xi, y)d\xi dy$. Using the fact that $\xi = r\cos\phi$, $y = r\sin\phi$ and $d\xi dy = rdrd\phi$, one obtains

$$p(r, \phi) = \frac{r}{2\pi\sqrt{a}\sigma^2}\exp\left[-\frac{(1 + c\sin^2\phi)r^2 - 2rA\cos\phi + A^2}{2a\sigma^2}\right]. \tag{9}$$

3.1 Phase probability density

The marginal pdf of the phase, $p(\phi)$, is obtained by integrating $p(r, \phi)$ over r. Upon completing square in r in the numerator of the exponent in (9), we have

$$p(\phi) = \frac{1}{2\pi\sqrt{a}\sigma^2}\exp\left[-\frac{A^2 a\sin^2\phi}{2a\sigma^2(1 + c\sin^2\phi)}\right]\int_0^\infty r\exp\left[-\frac{(r - b)^2}{2\sigma'^2}\right]dr, \tag{10}$$

where

(11) $$b = \frac{A\cos\phi}{1 + c\sin^2\phi} \ \text{and} \ \sigma'^2 = \frac{a\sigma^2}{1 + c\sin^2\phi}.$$

The $p(\phi)$ can be further written as

(12) $$p(\phi) = \frac{\sqrt{a}e^{-\rho/a}\left\{1 + \sqrt{\pi}\cdot\frac{b}{\sqrt{2}\sigma'}\left[1 + \mathrm{erf}\left(\frac{b}{\sqrt{2}\sigma'}\right)\right]\exp\left(\frac{b^2}{2\sigma'^2}\right)\right\}}{2\pi(1 + c\sin^2\phi)},$$

where erf() denotes the error function defined by [8]

(13) $$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt,$$

and

(14) $$\frac{b}{\sqrt{2}\sigma'} = \sqrt{\frac{\rho}{a}}\frac{\cos\phi}{\sqrt{1 + c\sin^2\phi}}.$$

The phase pdf has therefore been expressed in terms of the CNR and the AM signal parameters m and σ_m^2. It is an even function of ϕ. A plot of $p(\phi)$ *vs*. ϕ is shown in Figure 1 for five values of the CNR and for $m = 0.5$, $\sigma_m^2 = 0.04$. This value of σ_m^2 is sufficiently small that most of the modulating function fluctuations satisfy the usual restriction that $|g(t)| \leq 1$. If $m = 0$, then $c = 0$, $a = 1$, and (12) and (14) reduce to the phase pdf for a sinusoidal signal, as given in, for example, [1] [p. 140, equation (4-219)].

3.2 Envelope probability density

The marginal pdf of the envelope, $p(r)$, is obtained by integrating (9) over ϕ:

(15) $$p(r) = \frac{r}{2\pi\sqrt{a}\sigma^2}\int_0^{2\pi}\exp\left[-\frac{(1 + c\sin^2\phi)r^2 - 2rA\cos\phi + A^2}{2a\sigma^2}\right]d\phi.$$

An exact evaluation of this integral appears to be difficult. Instead, we obtain approximate expressions for $p(r)$ that are valid, for $r \ll r_0$ and for $r \gg r_0$, where r_0 is defined below. To proceed, we factor out the large-ρ behavior of $p(r)$. When $\rho \gg 1$, $r \approx A + z$. It follows that $p(r)$ can be approximated by an Gaussian function

(16) $$\exp[-\frac{(r-A)^2}{2a\sigma^2}],$$

where the constant terms are ignored. Then, we write (15) as

$$(17)\ p(r) = \frac{r}{\pi\sqrt{a}\sigma^2}\exp\left[-\frac{(r-A)^2}{2a\sigma^2}\right]\int_0^{\pi}\exp\left\{-\frac{cr[r\sin^2\phi + 2r_0(1-\cos\phi)]}{2a\sigma^2}\right\}d\phi,$$

where

$$(18)\qquad r_0 = \frac{A}{c} = \frac{\sigma}{\sqrt{2\rho}m^2\sigma_m^2},$$

and the integral in (15) over the interval $(0, 2\pi)$ has been written as twice the integral over $(0, \pi)$. For $r \ll r_0$, the contribution of $r\sin^2\phi$ in the integrand in (17) can be neglected compared with that of $2r_0(1-\cos\phi)$ for ϕ in $(0, \pi)$. The envelope pdf then becomes

$$(19)\qquad p(r) \approx \frac{r}{\sqrt{a}\sigma^2}\exp\left(-\frac{r^2+A^2}{2a\sigma^2}\right)I_0\left(\frac{Ar}{a\sigma^2}\right),$$

where $I_0()$ denotes the modified Bessel function of the first kind of order zero [8]. It is noted that if r and A are scaled by $\sqrt{a}$, then (19) is the Rice distribution [1] for the scaled variables.

For $\rho \ll 1$ and/or small signal parameters, $c \ll 1$, and the term $c\sin^2\phi$ in the integrand in (15) can be neglected compared with 1. The resulting $p(r)$ is identical to that in (19).

In terms of the normalized envelope, $r' \equiv r/\sigma$, ρ, and the signal parameters, (19) becomes

$$(20)\qquad p(r) \approx \frac{r'}{\sqrt{a}\sigma}\exp\left(-\frac{\rho}{a} - \frac{r'^2}{2a}\right)I_0\left(\frac{\sqrt{2\rho}r'}{a}\right) \quad r \ll r_0.$$

For $r \gg r_0$, the term $2r_0(1-\cos\phi)$ in the integrand in (17) can be neglected compared with $r\sin^2\phi$ only if ϕ is in the interval $(0, \pi/2)$. For ϕ in the interval $(\pi/2, \pi)$, as it approaches π, $r\sin^2\phi$ eventually becomes smaller than $2r_0(1-\cos\phi)$, even if $r \gg r_0$. This shows the need to split the integral into one over $(0, \pi/2)$ and another over $(\pi/2, \pi)$ and approximate each differently. The integral over $(0, \pi/2)$, after neglecting $2r_0(1-\cos\phi)$, is evaluated to be

$$(21)\qquad \int_0^{\pi/2}\exp\left(-\frac{cr^2\sin^2\phi}{2a\sigma^2}\right)d\phi = \frac{\pi}{2}\exp\left(-\frac{cr^2}{4a\sigma^2}\right)I_0\left(\frac{cr^2}{4a\sigma^2}\right).$$

The integral over $(\pi/2, \pi)$ becomes, after a simple change of integration variable and an elementary manipulation,

$$\text{(22)} \qquad \exp\left(-\frac{2Ar}{a\sigma^2}\right)\int_0^{\pi/2}\exp\left\{-\frac{cr[r\sin^2\phi - 2r_0(1-\cos\phi)]}{2a\sigma^2}\right\}d\phi.$$

In this form, the term $2r_0(1-\cos\phi)$ can now be neglected compared with $r\sin^2\phi$, and the resulting integral is the same as that in (21). The exponential multiplying the integral in (22) is small compared with 1. To see this, it is noted that, because of the Gaussian in (17), $p(r)$ is appreciable only for r in the neighborhood of A. The exponent $2Ar/(a\sigma^2)$ is therefore $\sim 2A^2/(a\sigma^2) = 4\rho/a$. It is shown in the next paragraph that $r \gg r_0$ corresponds to large ρ, for which $a \approx c$. Hence the exponent is $\sim 2/(m^2\sigma_m^2)$. In order for the modulating function to satisfy $|g(t)| \leq 1$ well, the upper bound on σ_m^2 should be about 0.1. Thus for m varying between 0.1 and 1, the exponent varies from about 2000 to about 20, showing that the result of (22) can be neglected compared with that in (21). Substituting the latter result in (17), one obtains

$$\text{(23)} \qquad p(r) \approx \frac{r}{2\sqrt{a}\sigma^2}\exp\left[-\frac{(r-A)^2}{2a\sigma^2}\right]\exp\left(-\frac{cr^2}{4a\sigma^2}\right)I_0\left(\frac{cr^2}{4a\sigma^2}\right).$$

For $r \sim A$, the argument of I_0 in (23) is $\sim \rho/2 \gg 1$. Replacing I_0 by its asymptotic behavior, $I_0(x) \approx e^x/\sqrt{2\pi x}$ for $x \gg 1$ [8], yields a $p(r)$ that is Gaussian, in agreement with one's expectation. In terms of r', ρ, and the signal parameters, $p(r)$ can be written as

$$\text{(24)} \qquad p(r) \approx \frac{r'}{2\sqrt{a}\sigma}\exp\left[-\frac{(r'-\sqrt{2\rho})^2}{2a}\right]\exp\left(-\frac{cr'^2}{4a}\right)I_0\left(\frac{cr'^2}{4a}\right) \qquad r \gg r_0.$$

The conditions $r \ll r_0$ and $r \gg r_0$ can be considered equivalent to low and high CNR, respectively. To see this, the maximum envelope, $r_{\max}$, beyond which $p(r)$ is negligible is first estimated. For small CNR, $p(r)$ is approximately Rayleigh, with the peak occurring at $r \approx \sigma$. One may choose $r_{\max}$ to be a multiple of σ, say, 5σ. For large CNR, $p(r)$ is approximately Gaussian, with the peak occurring at $r \approx A$ and a standard deviation of $\sqrt{a}\sigma$. One may estimate $r_{\max}$ to be the sum of A and a multiple, again say five, of the standard deviation, resulting in

$$\text{(25)} \qquad r_{\max} = A + 5\sqrt{a}\sigma = (\sqrt{2\rho} + 5\sqrt{a})\sigma.$$

Since the high-CNR estimate exceeds the low-CNR estimate, (25) is taken to be the maximum envelope for all CNRs. A comparison of $r_{\max}$ with r_0 is shown in Table 1 for four values of the CNR, $m = 0.5$, and $\sigma_m^2 = 0.04$. In Figure 2, $p(r)$ calculated from the approximate expressions are compared with that obtained by numerical integration, for the same four CNRs and signal parameters. In the absence of any expression to connect the $r \ll r_0$ and $r \gg r_0$ results, $p(r)$ is calculated from (20) for $r < r_0$ and from (24) for

Table 1: Comparison of $r_{\max}$ with r_0; $m = 0.5$, $\sigma_m^2 = 0.04$.

CNR (dB)	$r_{\max}/\sigma$	r_0/σ
0	6.46	70.71
10	9.95	22.36
20	22.80	7.07
30	67.63	2.24

$r > r_0$. This results, in general, in a discontinuity at $r = r_0$. The dashed curves in Figure 2 show the pdfs calculated in this way. The solid curves correspond to evaluating the integral in (17) numerically. For CNR of 0 dB, the entire envelope range is much smaller than r_0, and (20) agrees well with the numerically-integrated result. For CNR of 30 dB, the envelope range in which $p(r)$ is significant is much beyond r_0, and (24) gives a good description of the pdf. For CNR of 10 dB, although $r_{\max} < r_0$, it is a much larger fraction of r_0 than is the case for CNR of 0 dB. The agreement is less satisfactory, as one would expect. For CNR of 20 dB, $r_0 < r_{\max}$, but $r_{\max}/r_0$ is much smaller than is the case for CNR of 30 dB. The agreement is again less than satisfactory. An analysis of the fractional error between the approximate pdf and the numerically-integrated one confirms that the error increases as the envelope value approaches r_0 from either side.

4. CONCLUSIONS

In this memo, we have derived a closed-form pdf for the phase of amplitude modulated signals as a function of the CNR and the signal parameters. For the envelope pdf, approximate expressions are obtained for small and large CNRs. Using computer simulations, we showed that the numerical results agreed well with the theoretical derivations. Future efforts will be focused on applying the results for identification of radar and communication signals for EW purposes.

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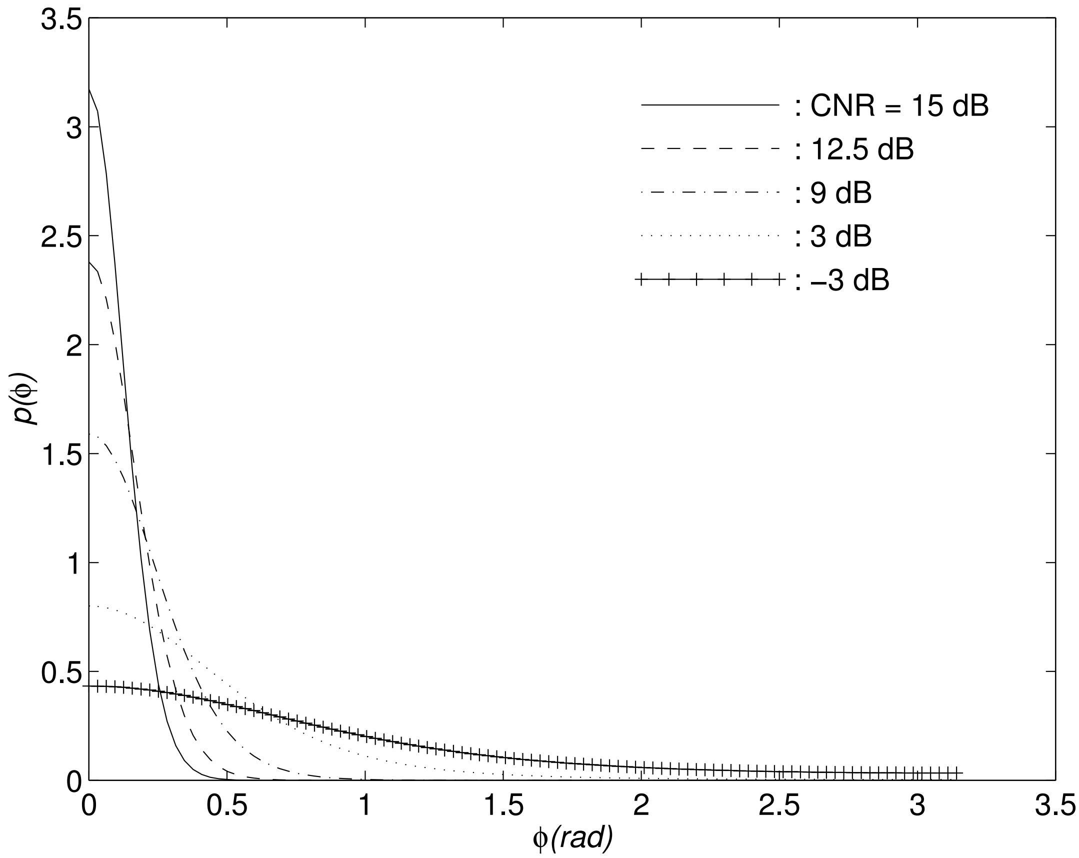

Figure 1: *Phase pdf of AM signal for five CNRs;* $m = 0.5$, $\sigma_m^2 = 0.04$.

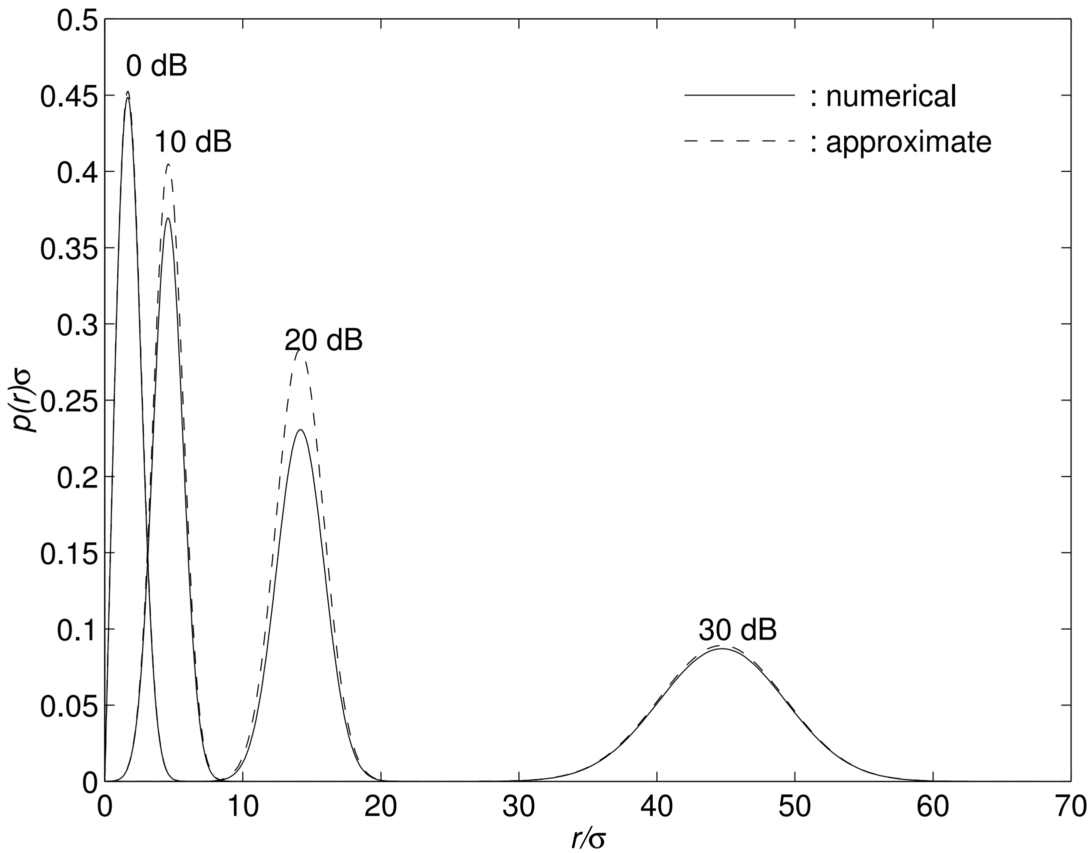

Figure 2: *Comparison of approximate AM envelope pdf with numerically-integrated results for four CNRs;* $m = 0.5$, $\sigma_m^2 = 0.04$.

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3. TITLE (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C,R or U) in parentheses after the title).

On the Phase and Envelope Distributions of Amplitude-Modulated Signals

4. AUTHORS (Last name, first name, middle initial. If military, show rank, e.g. Doe, Maj. John E.)

Sung, Stephen H. ; Zhou, Yifeng

5. DATE OF PUBLICATION (month and year of publication of document) March 2004	6a. NO. OF PAGES (total containing information. Include Annexes, Appendices, etc). 16	6b. NO. OF REFS (total cited in document) 8

7. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered).

Technical Memorandum

8. SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development. Include address).

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In this memo, the probability density functions (pdfs) for the phase and the envelope of an amplitude-modulated (AM) signal, where the modulation is modeled as a Gaussian random process, in the presence of narrow-band Gaussian noise are calculated. An exact, closed-form expression is obtained for the phase pdf, while approximate results, accurate for small and large carrier-to-noise ratios (CNRs), are derived for the envelope pdf.

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